TECHNICAL NOTE

Forced convection heat transfer about a cylinder placed in porous media with longitudinal flows

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Forced convection heat transfer about a circular cylinder placed in a saturated porous matrix with a uniform longitudinal flow has been studied both analytically and numerically. Based on Darcian flow and the boundary layer approximation, a closed form solution has been obtained. The Nusselt number is shown to be a function of a single parameter ξ, which is a measure of curvature. The predicted heat transfer results were compared with the complete two-dimensional solutions and available experimental data: the validity of the analytical prediction has been confirmed.

Keywords: porous media; forced convection; circular cylinder

Introduction

Heat transfer studies in a saturated porous layer have a wide range of applications from geophysics to various industrial operations.[1-] This report deals with the forced convection heat transfer from an isothermal cylinder with longitudinal flow. Despite its simplicity and importance in such industrial operations as reactors, absorbers, heat exchangers, and geothermal well design, it seems that no heat transfer correlation valid over a wide range of curvature is available. This paper, therefore, focuses on the effects of curvature on the heat transfer rate when the cylinder is subject to a longitudinal uniform flow. Darcy's law is used in formulating the momentum equation in the present analysis. The mathematical model is identical to that for liquid metal flow past a cylinder, where the velocity boundary layer is much thinner than the thermal boundary layer thickness; therefore, the velocity profile can be approximated with a slug-flow. This is the limiting case of the problems studied by Seban and Bond[4] as well as Kelly.[5]

Formulation

A semi-infinitely long cylinder is placed in a fluid saturated porous medium and subject to a uniform flow with velocity U_∞ and temperature T_∞ in the longitudinal direction, as shown in Figure 1. If it is assumed the physical properties of both the fluid and porous matrix do not vary with temperature and pressure, the nondimensional energy equation can be expressed as

$$\mathrm{Pe}_{r_0}\frac{\partial\theta}{\partial z}=\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}+\frac{\partial^2\theta}{\partial z^2} \qquad (1)$$

where Pe_{r_0}, θ, r, and z are defined by

$$\mathrm{Pe}_{r_0}=\frac{U_\infty r_0}{\alpha} \qquad \theta=\frac{T_*-T_\infty}{T_w-T_\infty} \qquad r=\frac{r_*}{r_0} \qquad z=\frac{z_*}{r_0} \qquad (2)$$

The corresponding boundary conditions are

$$\begin{aligned}&\theta=1 \quad \text{at } r=0\\ &\theta\to 0 \quad \text{as } r\to\infty\end{aligned} \qquad (3)$$

The inflow and outflow boundary conditions will be discussed later.

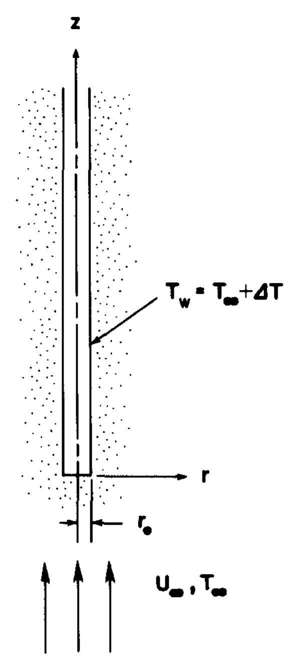

Figure 1 Cylinder placed in a homogeneous porous medium subject to uniform longitudinal flow

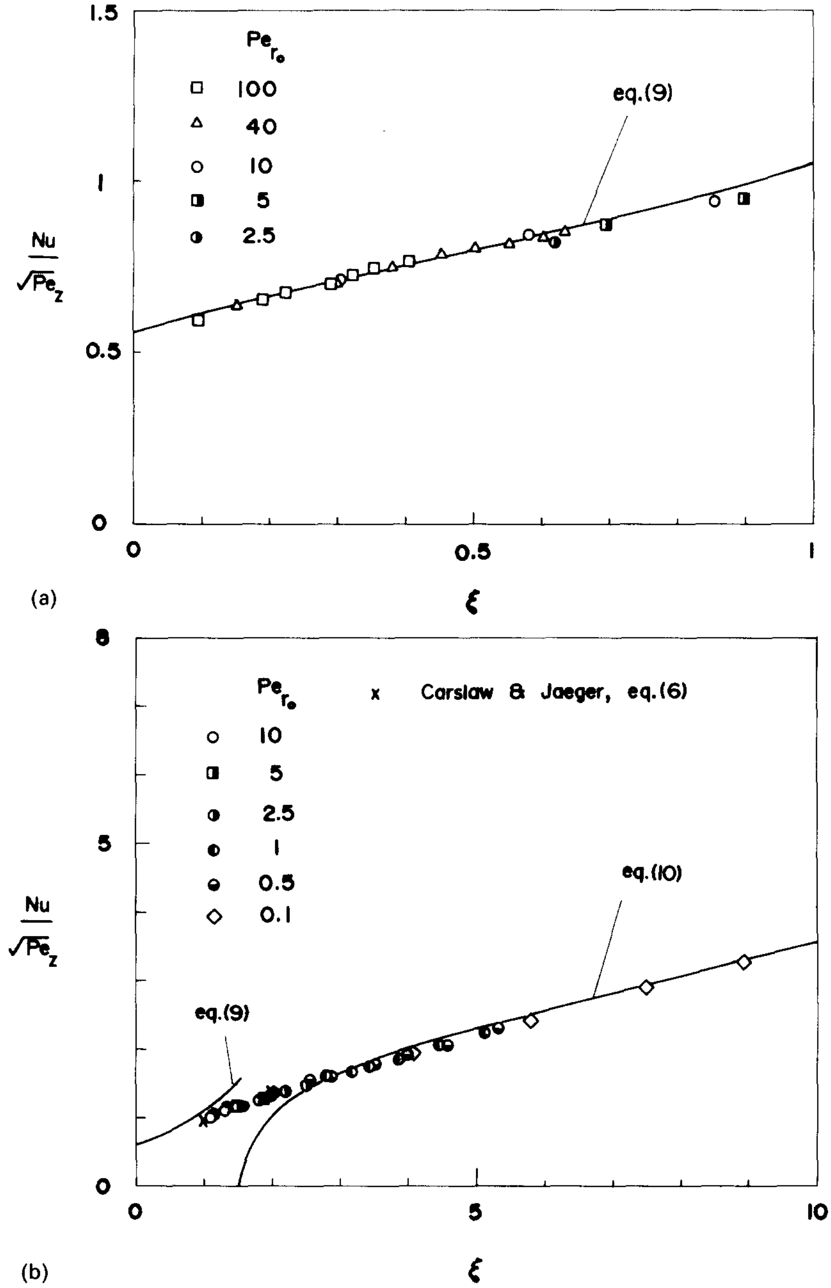

Figure 2 Dimensionless local heat transfer coefficient as a function of ξ
(a) $0<\xi<1$
(b) $1<\xi<10$

Notation

h	Heat transfer coefficient
J_0	Bessel function of the first kind of order 0
k	Effective thermal conductivity
Nu	Nusselt number
Pe	Peclet number
q	Heat flux per unit area
r	Radial coordinate
r_0	Radius of cylinder
T	Temperature
U	Fluid velocity
Y_0	Bessel function of the second kind of order 0
z	Longitudinal coordinate
γ	Euler's constant
θ	Dimensionless temperature
ξ	Dimensionless parameter

Subscripts

*	Dimensional quantity
∞	Condition at infinity
r_0	Characteristic length
w	Condition at wall
z	Characteristic length

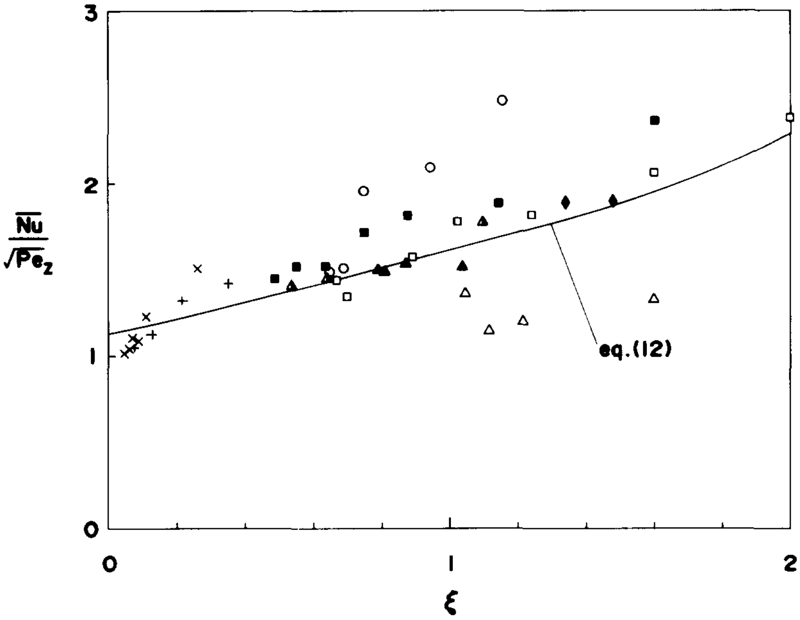

Figure 3 Average Nusselt number for small ξ compared with Gabor's experimental data. The porous materials are glass (○); copper (△, ▲, ▲, □, ■); aluminium (◆); cellulose acetate (X, +). The working fluid is air

Boundary layer approximations and heat transfer results

The boundary layer approximations are applied to Equation 1 to yield the following energy equation:

$$\mathrm{Pe}_{r_0}\frac{\partial\theta}{\partial z}=\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r} \tag{4}$$

The closed form solution[6] of Equation 4 with the boundary conditions (Equation 3) is

$$\theta=1+\frac{2}{\pi}\int_0^\infty e^{-\xi^2 r_0^2 u^2}\frac{J_0(ur_*)Y_0(ur_0)-Y_0(ur_*)J_0(ur_0)}{u(J_0^2(ur_0)+Y_0^2(ur_0))}\,du \tag{5}$$

The heat flux at the cylinder surface, therefore, can be obtained by the following equation:

$$q=h(T_\infty-T_w)=-k\left.\frac{\partial T_*}{\partial r_*}\right|_{r_*=r_0}$$
$$=\frac{4k(T_\infty-T_w)}{r_0\pi^2}\int_0^\infty e^{-\xi^2 r_0^2 u^2}\frac{du}{u\{J_0^2(ur_0)+Y_0^2(ur_0)\}}du \tag{6}$$

where ξ is defined by

$$\xi=\frac{z_*}{r_0\sqrt{\mathrm{Pe}_z}} \tag{7}$$

For small and large values of ξ, Equation 6 takes asymptotic forms. Since the local Nusselt number based on the longitudinal dimension is defined by

$$\mathrm{Nu}=\frac{hz_*}{k} \tag{8}$$

the respective equations for the local Nusselt number are

$$\frac{\mathrm{Nu}}{\sqrt{\mathrm{Pe}_z}}=\frac{\xi}{2}+\frac{1}{\sqrt{\pi}}+\left(\frac{\xi}{2}-\frac{1}{\sqrt{\pi}}\right)\sum_{n=1}^{\infty}\left(\frac{\xi}{2}\right)^{2n} \tag{9}$$

for small values of ξ, and

$$\frac{\mathrm{Nu}}{\sqrt{\mathrm{Pe}_z}}=\frac{\xi}{\ln 2\xi-\gamma}\left\{1-\sum_{n=1}^{\infty}\left(\frac{\gamma}{2(\ln(2\xi)-\gamma)}\right)^n\right\} \tag{10}$$

for large values of ξ. In Equation 10, $\gamma=0.57722\cdots$ is the Euler's constant. The first two terms in Equation 9 correspond to Gabor's heat transfer correlation.[3] Moreover, $\xi=0$ in Equation 9 leads to a well-known analytical result for the flat plate case.[7]

Discussion

The heat transfer results from Equations 9 and 10 are shown in Figures 2(a) and 2(b). Shown also are the results obtained by direct numerical solutions of Equation 1 for several values of Pe_{r_0}, which are indicated by the various symbols. The inflow and outflow boundary conditions used for the numerical calculations are

$$\theta \to 1 \quad \text{as } z\to-\infty$$
$$\theta_{zz}\to 0 \quad \text{as } z\to+\infty \tag{11}$$

The heat transfer correlation obtained by the respective asymptotic series diverges in the vicinity of $\xi=1$, but values taken from the figure in the book by Carslaw and Jaeger[6] corresponding to $\xi=1$ and 2 are marked with x points and illustrate the true nature of Equation 6 in that vicinity. Excellent agreement of the analytical solution with the numerical two-dimensional results is displayed in these figures and fully justifies the use of the boundary layer approximations and the resulting heat transfer correlation.

In Figure 3, the nondimensionalized average heat transfer results from Gabor's experiment are assembled to compare them with the present prediction. The average predicted Nusselt number for small values of ξ is

$$\frac{\overline{\mathrm{Nu}}}{\sqrt{\mathrm{Pe}_z}}=\frac{2}{\sqrt{\pi}}+\frac{1}{2}\xi-\frac{1}{6\sqrt{\pi}}\xi^2+\frac{1}{16}\xi^3-\cdots \tag{12}$$

Agreement with the experiment is generally good and again confirms the validity of the analysis. Some deviation may imply the breakdown of Darcian flow. In closing, it is concluded that the curvature effect on the heat transfer rate is indeed significant, and the amount of deviation from the flat plate case is shown to be a function of ξ.

Acknowledgement

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